X. *A Determination of "v," the Ratio of the Electromagnetic Unit of Electricity to the Electrostatic Unit.*

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THE experiments made by one of us in 1883 having given a value of "*v*" considerably smaller than the one found by several recent researches, it was thought desirable to repeat those experiments. The method used in 1883 was to find the electrostatic and electromagnetic measures of the capacity of a condenser; the electrostatic measure being calculated from the dimensions of the condenser, the electromagnetic measure determined by finding the resistance which would produce the same effect as that produced by the repeated charging of the condenser placed in one arm of a Wheatstone's Bridge. In the experiments of 1883 the condenser used in determining the electromagnetic measure of the capacity was not the same as the one for which the electrostatic measure had been calculated, but an auxiliary one, without a guard ring, the equality of the capacity of this condenser and that of the guard ring condenser being tested by the method given in MAXWELL'S 'Electricity and Magnetism,' vol. 1, p. 324.

In repeating the experiment we adopted at first the method used before, using, however, a key of different design for testing the equality of the capacity of the two condensers by MAXWELL'S method. We got very consistent results, practically identical with the previous ones. We may mention here, since it has been suggested that the capacity of the leads might account for the small values of "*v*" obtained, that this capacity is allowed for by the way the comparison between the capacities of the auxiliary and guard ring condensers is made, for the same leads are used both in this comparison and in the determination of the electromagnetic measure of the capacity of the auxiliary condenser; the capacity of the auxiliary condenser, plus that of its leads, is made equal to the capacity of the guard ring condenser, and it is the capacity of the auxiliary condenser, plus its leads, which is determined in electromagnetic measure. As the introduction of the auxiliary condenser introduced increased possibilities of error, we endeavoured to determine directly the electromagnetic measure of

the capacity of the guard ring condenser, by using a complicated commutator which worked both the guard ring and the condenser. At first we tried one where the contacts were made by platinum styles attached to a tuning fork, but as the results were not so regular as we desired, we replaced the tuning fork commutator by a rotating one driven by a water motor. A stroboscopic arrangement was fixed to this commutator so that its speed might be kept regular and measured. With this arrangement, which worked perfectly, we got values for the electromagnetic measure of the capacity of the condenser distinctly less than those obtained by the old method. We then endeavoured to find out the cause of this difference, and after a good deal of trouble discovered that in the experiments by which the equality of the capacities of the guard ring and auxiliary condensers was tested by MAXWELL'S method, the guard ring did not produce its full effect. When the guard ring of the standard condenser was taken off, and its capacity made equal by MAXWELL'S method to the capacity of the auxiliary condenser, the two methods gave identical results; but the effect of adding the guard ring was less in the old method than in the new. We found also, by calculation, that the effect produced by the guard ring in the old method was distinctly too small, while that determined by the new method agreed well with its calculated value. As the new method was working perfectly satisfactorily, and as it possesses great advantages over the old one, inasmuch as we get rid entirely of the auxiliary condenser, and can also alter the speed of the rotating commutator with very much greater ease and considerably greater accuracy than in any arrangement where the speed is governed by a tuning fork, we discarded the old method and adopted the new one which we now proceed to describe, beginning by considering the errors to which this method is liable.

Advantages of the Method of Determining "v."

The best way of discussing the advantages of this method is to consider the quantities which have to be measured and the accuracy which can be obtained in their measurement. The investigation naturally divides into two parts (1) the determination of the capacity of a condenser in electrostatic measure; (2) the determination of the capacity of the same condenser in electromagnetic measure. Let us begin by considering the first part. The condenser consisted of two co-axial cylinders, the inner cylinder being provided with a guard ring. If the distribution of electricity on the middle part of the inner cylinder were the same as that on an equal length, l, of an infinite cylinder whose radius is a, surrounded by a co-axial infinite cylinder of radius b, the electrostatic measure of the capacity would be $\frac{1}{2}\, l/\log b/a$. The actual case may differ from this ideal one in some or all of the following ways. (1) The two cylinders may not be quite co-axial; this, however, is not important if we know the distance betwen the axes, as we can find the capacity of the system got by placing one cylinder anywhere inside another. (2) The cross sections of the cylinders may

not be accurately circles. The effect on the capacity of a slight departure from circularity is calculated below, so that this effect may be corrected. (3) The conductors may not be true cylinders but swell or contract slightly as we proceed along their lengths; we show, however, below, how to correct for an effect of this kind. (4) The existence of the air space between the guard ring and the middle cylinder will cause the distribution of electricity near the ends of this cylinder to be irregular, and there will also be some electricity on the cross section of the cylinder; we have, therefore, found the distribution of electricity in a case so nearly resembling this as to allow us to use the result as a correction. In the arrangement we used the potential of the guard ring differed slightly from that of the middle cylinder, the very small correction due to this is, however, easily calculated. Since we know the corrections, the capacity of the condenser can be calculated in terms of its dimensions, and the only errors to which we are liable are those which may be made in the determination of these dimensions. The lengths which have to be measured with great accuracy are the length of the middle cylinder, its radius and that of the outer cylinder, and the distance between the cylinders. The first three of these are long enough to be measured by the ordinary methods of measuring length, without danger of an error greater than one part in 3000, the fourth, however, is too small to be measured with so great an accuracy by these methods, it was determined, therefore, by finding v, the volume of water required to fill the space between the two cylinders, then d the distance between the cylinders is given by the formula

$$d = \frac{v}{\pi l (a + b)}$$

where l is the length of the middle cylinder and a and b the radii of the two cylinders. In this way the percentage error of d was not greater than those of a, b, and l. Since an accuracy of one part in 3000 can be obtained in the measurements of the dimensions of the cylinders, and since the electrostatic measure of the capacity is of the dimensions of a length, this measure of the capacity can be obtained correct to one part in 3000.

We now pass on to the determination of the capacity in electromagnetic measure. This was determined by balancing, in a Wheatstone's bridge, a discontinuous current produced by rapidly charging the condenser against a steady current derived from the battery which charged the condenser. In order to calculate the electromagnetic measure of the capacity it is necessary to know accurately the number of times per second the condenser is charged, and to keep this number constant. The charging and discharging of the condenser were effected by a commutator driven by a Thirlmere Water Motor, the water being obtained, not from the main, but from a cistern at the top of the Laboratory. The number of revolutions per second made by the commutator was compared by a stroboscopic arrangement with the frequency of an electrically driven tuning fork. The observer (G.F.C.S.) was able, after practice, to govern the

speed of the commutator so efficiently that when the condenser was in action the spot of light reflected from the mirror of the galvanometer did not move over more than half a millimetre.

The accuracy of measurement of the number of times the condenser was charged per second is thus practically the same as the accuracy of the determination of the frequency of the tuning-fork ; this frequency could be determined (see *infra*) to less than one part in 10,000.

The limit which is practically put on the determination of the electromagnetic measure of the capacity of the condenser is that imposed by the galvanometer. With the galvanometer we employed, which was one made in the laboratory, having about 30,000 turns and a resistance of 17,400 legal ohms, when the resistance of the variable arm of the Wheatstone's bridge was 2500 ohms, an alteration of 2 ohms could be detected ; thus the measurement of the resistance equivalent to the repeatedly charged condenser could be made to one part in 1250 : an error of this magnitude would cause an error of one part in 2500 in the value of "v," and as all the other measurements were more accurate than this, there seems no reason why this method should not give as accurate a value of "v" as that obtained for the ohm.

The electromagnetic way of measuring the capacity affords us the means of testing the accuracy of the corrections applied to the electrostatic measure of the capacity ; we availed ourselves of this in the case of the correction for the effect of the air space between the middle cylinder and the guard-ring ; we altered the thickness of this air space and found that the effect of this alteration was accurately represented by the correction we employed. One great advantage of the method is the ease with which the number of times per second the condenser is charged can be altered ; this affords a valuable means of detecting any leakage or any effect due to self-induction.

Calculation of the Electrostatic Measure of the Capacity of the Condenser.

Description of the Condenser.—The condenser, which was designed some years ago by Lord RAYLEIGH, is represented in section in fig. 1, and in plan in fig. 2. BHPD is a thick ebonite board, placed in an approximately horizontal position ; in this board two concentric circular grooves are cut. A cylindrical brass ring, HP, whose external diameter is about 23 cm., and whose height is about 10 cm., fits into the smaller of these grooves. Three pieces of ebonite carefully ground down to the same thickness (about 3 mm. in most of the experiments), with V-shaped grooves cut in them to increase the distance over which the electricity would have to leak are placed at equal ntervals on the top of this ring. On these the brass cylinder FGMN is placed ; this cylinder is of exactly the same diameter as the cylindrical ring HP, and is about 60 cm. long. The cylinders GFMN and HP are placed so that their axes are coincident. On the top of this cylinder three pieces of ebonite similar to those on HP are placed, and

upon the top of these a cylindrical ring EL, similar to the ring at the bottom. Another brass cylinder, ABCD, made in three pieces, two rings somewhat similar in height to the rings HP, EL, and a long middle piece of the same length as the cylinder FGMN, is then fitted over the other cylinders, the bottom ring fitting into the outer groove in the ebonite board; the internal diameter of this cylinder is about 25 cm.

Fig. 1.

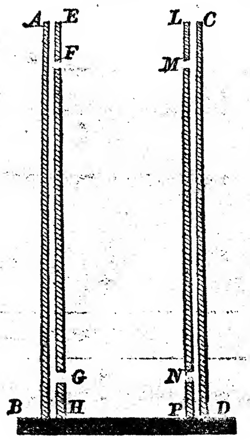
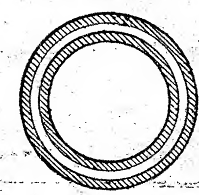


Fig. 2.



The cylinders are made co-axial by means of three pieces of ebonite worked down to the same thickness (the difference between the radii of the cylinders) pushed by rods attached to them down between the cylinders, the cylinders are adjusted until these three pieces of ebonite arranged symmetrically round the cylinder are each just in contact with the two cylinders; the rods were then removed. The insulation between the inner and outer cylinders and between the inner cylinder and its guard rings was tested by connecting one of these to earth, and the other to a charged gold leaf electroscope; the condenser was not used unless there was no appreciable loss of electricity shown by the electroscope in five minutes.

Calculation of the Capacity.—The capacity of the system regarded as two co-axial cylinders of circular section with a uniform distribution of electricity over them is

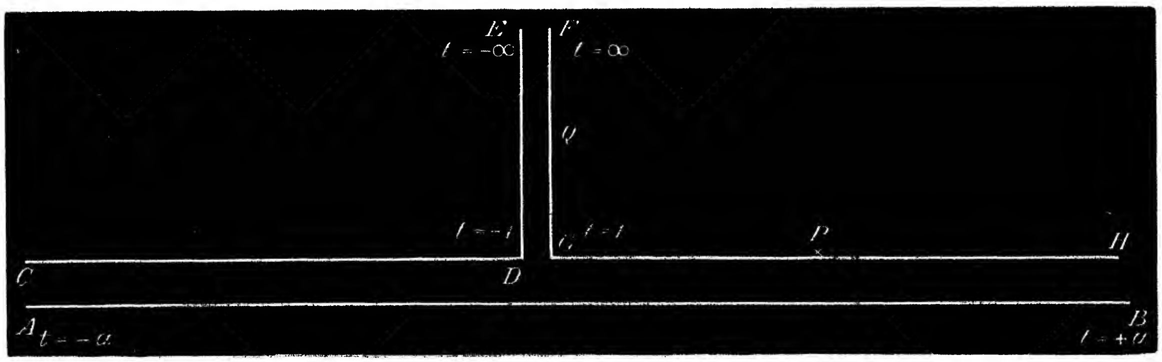
$$\tfrac{1}{2}\, l/\log\frac{a}{b},$$

where a is the radius of the outer cylinder, b that of the inner, and l the length of the cylinder FGMN.

Correction for want of coincidence between the Axes.—It is shown in a paper by J. J. Thomson, "On the Determination of the number of Electrostatic units in the Electromagnetic unit of Electricity" ('Phil. Trans.,' 1883, p. 714), that if c be the small distance between the axes of the cylinders, the capacity is

$$\frac{1}{2}\frac{l}{\log\frac{a}{b}}\left\{1+\frac{c^2}{(a^2-b^2)\log\frac{a}{b}}\right\}.$$

Correction for the want of equality in the distribution produced by the air spaces between the inner cylinder and the guard rings.—To find this correction we shall find the distribution of electricity on the system represented in the figure, when AB is the section by the plane of the paper of an infinite horizontal metal plane, and CDE, FGH sections of conductors, CD and GH being horizontal and at the same distance from AD; and DE and FG vertical. Let h be the distance between the planes CD, and AB, and $2c$ the breadth of the slit DE FG.



Let us take AB as the axis of x and the vertical line midway between ED and FG as the axis of y. Then writing z for $x + \iota y$, and supposing that ϕ and ψ are the stream and potential functions respectively, we find by using SCHWARZ's method that the solution of the problem is given by the equations

$$dz = -\mathrm{A}\frac{\{1 - t^2\}^{\frac{1}{2}}}{t^2 - a^2}\,dt. \quad a < 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\phi + \iota\psi = \mathrm{B}\log\frac{t - a}{t + a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

t being supposed to have all real values from $-\infty$ to $+\infty$.

For putting $t = \sin\theta$, $a = \sin\alpha$, and integrating (1) we find

$$z = \mathrm{A}\left(\theta - \tfrac{1}{2}\cot\alpha\log\frac{\sin(\alpha - \theta)}{\sin(\alpha + \theta)}\right)$$

or

$$x + \iota y = \mathrm{A}\left(\theta - \tfrac{1}{2}\cot\alpha\log\frac{\sin(\alpha - \theta)}{\sin(\alpha + \theta)}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

as θ goes from 0 to α, the right hand side of this equation is real so that $y = 0$ and x ranges from 0 to $+\infty$, this gives the positive half of the plane AB. As θ goes from α to $\frac{1}{2}\pi$

$$x + \iota y = \mathrm{A}\left(\theta - \tfrac{1}{2}\cot\alpha\log\frac{\sin(\theta - \alpha)}{\sin(\alpha + \theta)} + \tfrac{1}{2}\,\iota\pi\cot\alpha\right)$$

so that $y = \frac{1}{2} \mathrm{A}\pi \cot \alpha$, and x ranges from ∞ to $\frac{1}{2} \mathrm{A}\pi$, so if

$$h = \frac{1}{2} \mathrm{A}\pi \cot \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

$$c = \frac{1}{2} \mathrm{A}\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

this will give the portion GH of the diagram. When $\sin \theta$ is greater than unity we may put

$$\theta = \frac{1}{2} \pi + \iota \vartheta,$$

the right hand side of (3) now equals

$$\mathrm{A} \left\{ \frac{\pi}{2} + \iota \vartheta - \frac{1}{2} \cot \alpha \log \frac{\cos \alpha (\epsilon^{\vartheta} + \epsilon^{-\vartheta}) + \iota \sin \alpha (\epsilon^{\vartheta} - \epsilon^{-\vartheta})}{\cos \alpha (\epsilon^{\vartheta} + \epsilon^{-\vartheta}) - \iota \sin \alpha (\epsilon^{\vartheta} - \epsilon^{-\vartheta})} + \frac{1}{2} \iota\pi \cot \alpha \right\}$$

calling the quantity under the logarithm $\mathrm{P} + \iota\mathrm{Q}$, we see since

$$\mathrm{P} + \iota\mathrm{Q} = \frac{\cos \alpha (\epsilon^{\vartheta} + \epsilon^{-\vartheta}) + \iota \sin \alpha (\epsilon^{\vartheta} - \epsilon^{-\vartheta})}{\cos \alpha (\epsilon^{\vartheta} + \epsilon^{-\vartheta}) - \iota \sin \alpha (\epsilon^{\vartheta} - \epsilon^{-\vartheta})}$$

that

$$\mathrm{P} - \iota\mathrm{Q} = \frac{\cos \alpha (\epsilon^{\vartheta} + \epsilon^{-\vartheta}) - \iota \sin \alpha (\epsilon^{\vartheta} - \epsilon^{-\vartheta})}{\cos \alpha (\epsilon^{\vartheta} + \epsilon^{-\ }) + \iota \sin \alpha (\epsilon^{\vartheta} - \epsilon^{-\vartheta})};$$

multiplying these together we see that $\mathrm{P}^2 + \mathrm{Q}^2 = 1$. So that $\log (\mathrm{P} + \iota\mathrm{Q})$ is wholly imaginary. Thus we see that as ϑ ranges from 0 to ∞, and t therefore from 1 to ∞, $x = \frac{1}{2}\mathrm{A}\pi$ and y ranges from $\frac{1}{2}\mathrm{A}\pi \cot \alpha$ to ∞, thus this range of values of t gives the portion GF of the figure. Since the real part of the right hand side of the equation (3) changes sign with θ or t, we see that the other portions of the figure are given by the negative values of t.

Since

$$\phi + \iota\psi = \mathrm{B} \log \frac{t - a}{t + a}$$

we see that as long as t is between $-a$ and a, that is for the portion AB of the figure,

$$\phi + \iota\psi = \iota\pi\mathrm{B} + \text{real quantities},$$

so that $\psi = \pi\mathrm{B}$, and, therefore, the potential is constant over AB; when t is not between these values, that is for the other portion of the figure $\mathrm{B} \log \{(t - a)/(t + a)\}$ is real, and therefore $\psi = 0$.

Thus these equations give us the solution of the problem when AB is maintained at the potential $\pi\mathrm{B}$ and CDE, FGH are at zero potential.

The quantity of electricity on the conductor FGP when P is a point on GH

$$= \frac{1}{4\pi}(\phi_{\text{P}} - \phi_{\text{F}})$$

$$= \frac{\text{B}}{4\pi} \log \frac{t - a}{t + a},$$

where t is the value of t at P. If we represent the increase in the quantity of electricity, due to the irregularity of the distribution, by supposing a strip of breadth d to be added to the conductor GH, and the distribution of electricity to be regular, and the same as if the air space were not present; the equation to find d is, if x is the value of x at P and V, the difference of potential between AB and GH.

$$\frac{\text{V}}{4\pi h}\{x - c + d\} = -\frac{\text{B}}{4\pi} \log \frac{t - a}{t + a},$$

substituting for x and t their values in terms of θ, and remembering that

$$\text{V} = \pi\text{B}, \qquad c = \tfrac{1}{2}\text{A}\pi \qquad h = \tfrac{1}{2}\text{A}\pi \cot \alpha$$

we get

$$\text{A}\left(\theta - \tfrac{1}{2}\cot\alpha \log \frac{\sin(\theta - \alpha)}{\sin(\alpha + \theta)}\right) - c + d = -\tfrac{1}{2}\text{A}\cot\alpha \log \frac{(\sin\theta - \sin\alpha)}{(\sin\alpha + \sin\theta)};$$

or

$$d = c\left\{1 - \frac{2}{\pi}\theta\right\} - \frac{h}{\pi}\log \frac{(\sin\theta - \sin\alpha)\sin(\alpha + \theta)}{(\sin\alpha + \sin\theta)\sin(\theta - \alpha)}.$$

Now if P be some distance from G we may put $\theta = \alpha$ and we get

$$d = c\left\{1 - \frac{2}{\pi}\alpha\right\} - \frac{h}{\pi}\log\cos^2\alpha,$$

from equations (4) and (5) we see that $\tan\alpha = c/h$, so that

$$d = c\left\{1 - \frac{2}{\pi}\tan^{-1}\frac{c}{h}\right\} + \frac{1}{\pi}h\log\left\{1 + \frac{c^2}{h^2}\right\}.$$

To deduce the corresponding solution for the cylinders from this we must multiply by the correction for curvature $1 + \frac{1}{4}h/a$, where a is here the radius of the inner cylinder, so that we have finally, if D be the whole breadth to be added for the two air spaces,

$$\text{D} = 2\left[c\left\{1 - \frac{2}{\pi}\tan^{-1}\frac{c}{h}\right\} + \frac{1}{\pi}h\log\left(1 + \frac{c^2}{h^2}\right)\right]\left(1 + \frac{1}{4}\frac{h}{a}\right).$$

Now in our condenser l was about 60, $2c = {\cdot}3$, and $h = 1$, so that if we put $\text{D} = 2c$ the value of the capacity will be correct to 1 part in 2000.

Correction for a small difference of potential between the guard ring and the middle cylinder.—The arrangement we used necessitates the existence of a small difference of potential between the cylinder and the guard ring, a slight modification of the preceding investigation will enable us to find the correction for this. If we put

$$\phi + \iota\psi = \frac{V}{\pi}\log\frac{t-a}{t+a} + \frac{\delta V}{\pi}\log(t+a),$$

then the potential over FGH = 0, that over EDC = δV, and over AB = V.

The breadth of the strip which must be added to compensate for the electricity on the portion QGH due to the difference of potential δV between CD and GH is

$$\frac{h}{\pi}\frac{\delta V}{V}\log\frac{(t_Q + a)}{(t_H + a)}.$$

Now, if QG is large,

$$GQ = A\log\frac{2t_Q}{\epsilon},$$

where ϵ is the base of the Napierian logarithms. Hence, since $t_H = a = \sin\alpha$, the breadth of the additional strip is

$$\frac{h}{\pi}\frac{\delta V}{V}\left\{\frac{QG}{A} - \log\frac{4\sin\alpha}{\epsilon}\right\};$$

but $A = c\,2/\pi$ and $\sin\alpha = c/h$ approximately, hence the breadth of the strip for the two guard rings is

$$h\frac{\delta V}{V}\left\{\frac{QG}{c} - \frac{2}{\pi}\log\frac{4c}{h\epsilon}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In our experiments $h/c = 6{\cdot}6$, QG = 1, so that the correction amounts to a strip whose breadth is about

$$7{\cdot}5\,\frac{\delta V}{V}.$$

The value of $\delta V/V$ depended on the speed, the usual value was about $\frac{1}{183}$. In this case the breadth of the strip would be about $\frac{1}{30}$ of a centimetre, and, since the length of the cylinder was about 60 cm., the correction amounts to about 1 part in 1800.

To test the accuracy of these corrections, determinations of the capacity of the condenser were made when the top guard ring was separated from the middle cylinder (1) by pieces of ebonite ·504 cm. thick, (2) by pieces ·067 cm. thick. The capacity of the cylinders with the thick ebonite was greater than that with the thin by about 11 parts in 2760. According to the results we have obtained for these

corrections the effect of increasing the thickness of the ebonite would be to add a breadth ·218 cm. to the cylinder in consequence of the increased air space, and ·06 in consequence of the difference of potential, thus the two would add ·28 to the length of the cylinder, and would increase the capacity by $\frac{\cdot 28}{61} \times 2760$, or 12 parts in 2760. Thus the observed and calculated results agree well together.

Correction for ellipticity of the cross section.—Let us consider the case of a cylinder whose cross section is represented by the equation

$$r = b\,\{1 + e\cos 2\theta\},$$

placed inside one whose cross section is represented by

$$r = a\,\{1 + \alpha\cos 2\theta + \beta\sin 2\theta\},$$

where, since the measurements of the cylinder show that e, α, β are less than $\frac{1}{2000}$, we can neglect the squares of these quantities.

Let the potential between the cylinders be given by

$$\mathrm{V} = \mathrm{A}\log r + \frac{\mathrm{C}\cos 2\theta}{r^2} + \mathrm{D}r^2\cos 2\theta + \frac{\mathrm{E}\sin 2\theta}{r^2} + \mathrm{F}r^2\sin 2\theta.$$

Then, neglecting the squares of e, α, β, the difference of potential between the cylinders is

$$\mathrm{A}\log\frac{a}{b},$$

and to the same approximation the charge per unit length is $\frac{1}{2}\mathrm{A}$, thus the capacity per unit length is $\frac{1}{2}\log a/b$. Here a and b are the means of any two radii of the cylinders at right angles to one another. If we take these values as the radii of the cylinders the only correction required will be one of the order of one part in $(2000)^2$, which may be neglected.

Correction of Conicality.—We may see how to get rid of this correction by considering the electrical distribution on two infinite conductors, the one a plane perpendicular to the axis of y, the other a corrugated plane represented by the equation

$$y = h + \beta\sin\frac{2\pi x}{l},$$

the other plane being taken as the plane of xz. Let V the potential between the planes be given by

$$\mathrm{V} = \mathrm{A}y + \mathrm{C}\sin\frac{2\pi x}{l}\,\{\epsilon^{2\pi y/l} - \epsilon^{-2\pi y/l}\}$$

putting $y = h + \beta\sin 2\pi x/l$, and making the potential constant and equal to V_0

$$\left.\begin{aligned} V_0 &= Ah \\ C &= -\frac{A\beta}{\epsilon^{2\pi h/l} - \epsilon^{-2\pi h/l}} \end{aligned}\right\} \text{neglecting } \beta^2.$$

Thus σ, the surface density on the plane of xz,

$$= \frac{V_0}{4\pi h}\left\{1 - \frac{\beta \sin\frac{2\pi x}{l}\frac{2\pi}{l}}{\epsilon^{2\pi h/l} - \epsilon^{-2\pi h/l}}\right\}$$

$$= \frac{V_0}{4\pi h}\left\{1 - \frac{(y-h)\frac{2\pi}{l}}{\epsilon^{2\pi h/l} - \epsilon^{-2\pi h/l}}\right\}.$$

Thus, if we choose h so that it is the *mean* distance between the plates, for the breadth on which we wish to find the charge, the second term will vanish in our integration, and we get for Q the quantity of electricity on a breadth x

$$Q = \frac{V_0}{4\pi h}x.$$

Thus we can use the ordinary formula even when the plates are slightly inclined, provided h is the mean distance. Any correction to this will be the order of the square of the inclination at least, and in our case may be neglected.

Measurement of Dimensions of Condenser.

The dimensions are all referred to the standard metre of the Cavendish Laboratory which has been compared with the standard of the Board of Trade. The errors of the divisions are too small to affect the measurements given below. The comparison of the lengths with the standard metre was made by means of a pair of reading microscopes with micrometer screws. The pitch of the screws is accurately $\frac{1}{50}$th of an inch, and the head of the screw is divided into 100 parts, so that one division of the screw-head corresponds to ·0002 inch. The tenths of divisions are easily read and are recorded. The screws were tested by Mr. FITZPATRICK when working with Mr. GLAZEBROOK at the Specific Resistance of Mercury, and were found to be free from sensible error in either pitch or uniformity.

The standard metre is correct at 0° C., and its temperature coefficient is ·000017 per 1° C.

We require the dimensions of the condenser at 16° C. The metal of which the condenser is made is much the same as that of the standard metre, so that if we assume that the temperatures of the condenser and standard metre are the same at

the time of comparison we shall simply have to correct the metre to 16° C. The temperature of the room never differed from 16° by more than 2°, so that no appreciable error can be introduced on this account.

External Diameter of Inner Cylinder.

The sliding calipers of the laboratory were used to measure this. The bar of the calipers rested on the flat top of the cylinder, so that the calipers could be moved backwards and forwards along the top. The jaws are supposed to be at right angles to the bar along which the sliding one moves, but this was found not to be exactly the case. To obviate this difficulty a small piece of brass was fastened to the end of one jaw, so that the contacts were made at the ends of both jaws. The calipers were then placed under the microscope and two definite marks read off. The standard metre was then placed beneath the microscopes and treated in the same way. The distance between the marks when the jaws of the calipers were in contact was determined by the micrometer screw alone.

The readings of the screws are given in terms of $\frac{1}{5}$ inch.

MAXIMUM Diameter of top end of Cylinder.

	Calipers.		23·8 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
1	1·1765	·6643	1·2308	·5970
2	1·1930	·6420	1·1939	·6280
3	1·8100	·5642	1·7980	·5648
4	1·8676	·5134	1·8793	·4965

The numbers in (4) are the mean of three readings.

These measurements gave as the distance between the marks—

	23·8 cm.	−·00260 in.	(1)
	,, ...	−·00262 ,,	(2)
	,, ...	−·00220 ,,	(3)
	,, ...	−·00104 ,,	(4)
Mean	23·8 cm	−·00211 in.	
		= 23·7946 cm.	

Distance between the marks with jaws of calipers closed. Read with left-hand screw—

(1.)	1·2779	(2.)	1·5778	(3.)	2·0561
	·7538		1·0430		1·5192
	·5241		·5348		·5369
(4.)	2·3553	(5.)	2·2183	(6.)	2·2187
	1·8136		1·6891		1·6895
	·5417		·5292		·5292
(7.)	2·3550	(8.)	2·5410	(9.)	2·5330
	1·8172		2·0096		1·9998
	·5378		·5314		·5332

The mean of these is

$$\frac{\cdot 53314}{5} \text{ in.} = \cdot 10662 \text{ in.} = \cdot 2708 \text{ cm.}$$

Thus we find that the maximum diameter at the top of the cylinder is

$$23\cdot 7946 - \cdot 2708 = 23\cdot 5238 \text{ cm.},$$

referred to the standard at 16° C.

To reduce the standard to 0° C we must multiply by $(1 + 16 \times \cdot 000017)$ and we get as the true diameter

$$23\cdot 5302 \text{ cm.}$$

Minimum diameter of top end of Cylinder.

	Calipers.		23·8 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
1. (Mean of 4 readings)	2·2158	0·4338	2·1822	0·4279
2. (Mean of 4 ,,)	1·9271	0·7133	1·8625	0·7406

Giving as minimum diameter of top end

(1.) 23·8 cm. — ·2708 cm. — ·00790 in.
(2.) 23·8 cm. — ·2708 cm. — ·00746 in.

Correcting for temperature, we find

Minimum diameter of top end = 23·5161 cm.

BOTTOM end of Cylinder.

MAXIMUM DIAMETER.

	Calipers.		23·8 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
1. (Mean of 4 readings)	1·4078	1·2042	1·4171	1·1950
2. (Mean of 4 ,,)	1·5469	1·0671	1·5910	1·0196

MINIMUM DIAMETER.

	Calipers.		23·8 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
1. (Mean of 4 readings)	1·6915	·9584	1·6814	0·9318
2. (Mean of 4 ,,)	1·6916	·9581	1·6495	0·9634

Thus, maximum diameter of bottom end equals

$$(1.)\ 23{\cdot}8 \text{ cm.} - {\cdot}2708 \text{ cm.} + {\cdot}00002 \text{ in.}$$
$$(2.)\ 23{\cdot}8 \text{ cm.} - {\cdot}2708 \text{ cm.} - {\cdot}00068 \text{ in.}$$

Correcting for temperature, we find

Maximum diameter of bottom end = 23·5348 cm.

The minimum diameter of bottom end equals

$$(1.)\ 23{\cdot}8 \text{ cm.} - {\cdot}2708 \text{ cm.} - {\cdot}00734 \text{ in.}$$
$$(2.)\ 23{\cdot}8 \text{ cm.} - {\cdot}2708 \text{ cm.} - {\cdot}00736 \text{ in.}$$

Correcting for temperature

Minimum diameter of bottom end = 23·5169 cm.

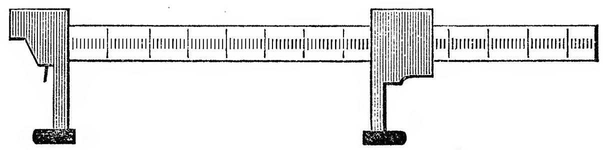
Collecting these results we have for the inner cylinder

Top end .	Maximum diameter	= 23·5302.
	Minimum diameter	= 23·5161.
Bottom end	Maximum diameter	= 23·5348.
	Minimum diameter	= 23·5169.
Mean of these		= 23·5245.

The corresponding ends of the measured diameters were found to be almost exactly on the same generating line, so that though the cylinder is slightly elliptical and conical, it is free from anything of the nature of helicality.

Measurement of the Internal Diameter of the Outer Cylinder.

This was found to be a good deal more troublesome than the measurement of the external diameter of the inner cylinder, the plan finally adopted was to fix two pieces of hardened steel to the ends of the jaws of the sliding calipers, thus



One side of each piece was polished, and the end was then ground and polished on a fine oilstone so as to form a good edge with the polished face. The shape of the end was semi-circular. In this way the edges made contact with the cylinder, and the cross wires (one of which was set carefully perpendicular to the line of travel of the microscopes) could be easily focussed on to the end of the steel.

It was not found practicable to determine exactly when contact was made in the same way as was done for the inner cylinder, since when the calipers were set to nearly the size of the cylinder scarcely any movement was possible. The sharp edges were also an impediment to the motion. We found, however, that by insulating one of the steel contact pieces we could determine accurately by the aid of a telephone when contact was made. As the cylinder was found to be nearly circular, and the formula for a slightly elliptical cylinder outside a circular one indicates that the lengths of two diameters at right angles to each other are required, two such diameters were measured. The following are the details of the measurements, each of the numbers being the mean of four observations:—

Top end.

Diameter A.

	Calipers.		25·4 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
(1)	1·3821	1·1721	1·3859	1·1798
(2)	1·3319	1·2060	1·3255	1·2324
(3)	1·1822	1·3445	1·1462	1·3993

Diameter B.

	Calipers.		25·4 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
(1)	1·1701	1·3843	1·1793	1·3787
(2)	1·1873	1·3762	1·1614	1·4032
(3)	1·1268	1·4415	1·1150	1·4485
(4)	1·8703	0·6891	1·8875	0·6620

Bottom end.

Diameter A.

	Calipers.		25·4 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
(1)	1·4352	1·0999	1·4289	1·1209
(2)	1·4734	1·0699	1·4918	1·0586

Diameter B.

	Calipers.		25·4 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
(1)	1·6109	0·9270	1·6131	0·9385
	1·6978	0·8427	1·7106	0·8371

Taking the mean of these and correcting for temperature we find

Top end . Diameter A = 25·4154 cm.
Diameter B = 25·4056 cm.

Bottom end Diameter A = 25·4125 cm.
Diameter B = 25·4122 cm.

Mean of these . . . . = 25·4114 cm.

Measurement of the Length of the Cylinder.

The length of the cylinder was transferred from the cylinder to the reading microscopes by means of the beam compasses of the laboratory; care being taken to keep the bar of the compasses parallel to the length of the cylinder while setting the compasses to the length of the cylinder.

On account of the length of the cylinder it was found difficult to ascertain by

moving the beam compasses just when contact was complete. A small piece of thin sheet steel (about ·03 cm. thick) was interposed between the end of the cylinder and the point of the beam compasses. The compasses were considered adjusted when a slight resistance to the motion of the feeling piece was perceived. The beam compasses were then placed under the microscopes, and the distance between two definite marks on their points determined. The points were then placed close together, so that the same resistance to the motion of the sliding piece was felt as in the former case. The distance between the marks was then ascertained by means of one of the microscopes and its screw. The distance being so small it seems unnecessary to compare it with the divisions of the standard metre.

DISTANCE between the Marks when the Compasses were Closed.

	Right-hand mark.	Left-hand mark.	
(1)	2·3156	1·7220	Mean of 4 observations.
(2)	2·1283	1·5308	,, 4 ,,
(3)	2·1287	1·5290	,, 5 ,,
(4)	2·9120	2·3154	,, 6 ,,

Giving, as the distance between the marks, ·11937 in., or ·3032 cm.

DISTANCE between the Marks when the Compasses were Open.

Calipers.		61·3 cm.		
Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.	
2·1297	·5698	2·0656	·5532	Mean of 4 observations.
2·4270	·2537	2·3819	·2415	,, ,,

Giving as the distance betwen the marks when open, 61·3 cm. — ·0138 in.

Hence the length of the cylinder when corrected for temperature equals 60·9784 cm.

The Distance between the Inner and Outer Cylinders.

Since the difference of the mean diameters is only about 1·09 cm., and since, on account of the difficulties of measurement and the irregularities in the shape of the cylinders, it is impossible to arrive at any satisfactory result by subtracting the mean diameter of one cylinder from that of the other, we had to apply some other method. We adopted that used in the experiment of 1883, which was to ascertain the amount of water required to fill the space between the two cylinders This amount was determined by weighing. The water employed was distilled, and was boiled a few hours previous to its use to enable it to absorb air bubbles more readily.

A 500 c.c. flask was filled with water and weighed, its contents were then transferred to the cylinders, and it was then weighed again. The difference in weight gives the weight of water transferred to the cylinder. This process was repeated until the space between the cylinders was full.

The weights of the full and empty flasks were determined to 1 centigrm.

The 500 grm. weight used to balance the water was compared with the standard 500 grm. weight of the Laboratory and found too heavy by ·055 grm. This has been allowed for. The equality of the arms of the balance was also tested.

The two cylinders were fastened down to a flat metal plate with a thin layer of cement so as to be quite water-tight. To get any accurate estimate of the volume of water required to fill the space it was necessary to provide some means to ascertain when the space was exactly full. The effects of capillarity, grease, &c., preclude any very accurate result being obtained when there is no top or cover fitted to the top of the cylinder, and as it was necessary to see whether any air bubbles were left inside a glass top had to be used. Two holes were bored through the glass, and tubes were fixed into these. The water was introduced through one tube, and the air escaped through the other. Although no difficulty was experienced in making the joint at the bottom quite water-tight with any of the cements employed, it took us several days to make a satisfactory joint at the top. The ease with which tightness at the bottom was secured was probably owing to the great weight of the cylinders.

The top gave us all the more trouble, because we could not tell whether it was watertight or not until we had almost completed the filling in of the water. If, then, the joint proved bad the whole of the time spent in weighing the water poured in was wasted.

We tried Prout's elastic glue, then gutta-percha dissolved in benzene, but both of these failed. The water seemed to loosen the hold of the glue upon the glass, so that although the system seemed air-tight it would not remain water-tight for more than a minute or two.

We finally tried some red wax which had been sent to the Laboratory by Professor THRELFALL, who obtained it in Germany, and this answered very well. It looks somewhat like a mixture of bees'-wax and sealing wax, and as it never gets quite hard it never cracks. It possessed another property which was also useful for our purpose, viz., that of melting at a comparatively low temperature. To apply the other cements in a satisfactory manner the glass had to be heated to a somewhat high temperature, and this frequently cracked it.

The wax when melted became very fluid, so that only an extremely thin layer was included between either the top or bottom plate and the cylinders.

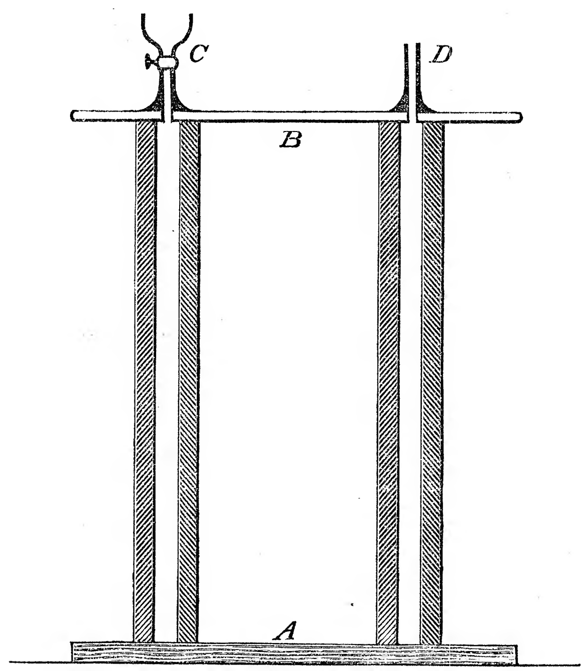
At first the water was poured in through a funnel inserted into one of the tubes, but with this arrangement it was found impossible to get rid of the last air bubble, since fresh quantities of air were continually carried down the tube. After some experiments with different arrangements for filling, we finally adopted as the means

of introducing the water a tube with a funnel top provided with a tap. The funnel could always be kept partially full of water by means of the tap, so that no air was introduced. The cylinders were slightly tilted so that the exit tube was at the highest place. As the last few grammes were poured in the air was gradually swept along by the advancing water and driven up the exit tube. The small bubble which sometimes remained under the exit tube was easily removed by agitating it by a fine wire introduced through the exit tube.

The amount of water remaining in the tap and the amount which rose up the exit tube were ascertained afterwards by detaching the tap and tube, and finding the weight of water required to fill them to the same extent as when the cylinders were filled.

While the water was still about 3 cm. from the glass the air was exhausted from the cylinders by a water pump; the pressure of the air was reduced to about 50 mm. of mercury. This had the effect of removing a good many bubbles, and we may hope that the experiment was free from error in this respect. The wax employed for fixing the top on to the cylinders stood this difference of pressure perfectly. A drying tube was placed between the cylinders and the tube leading to the pump in order to catch any water which might be carried off in the pumping. The increase in weight of the drying tube was found to be not more than ·02 or ·03 gramme, and this has been allowed for. The weighings have also been corrected to a vacuum.

The annexed sketch shows a section of the cylinders by a plane through their axis, and through the tap and exit tube.



A is the flat metal plate.
B the glass plate.
C the funnel and tap.
D the exit tube.

The following are the results of the weighings on two separate days :—

	[1.] Temp. 17.	[2.] Temp. 15·3.
Weight of water put into funnel	4403·15	4403·53
Weight of water left in tap and tube	1·33	1·35
Volume of a piece of wax underneath the glass top	·70	0
Weight of water in space between the cylinders is therefore	4402·52	4402·18
Correction to vacuum	4·66	4·66
Correction for temperature	5·10	3·91
Correction for error in 500-grm. weight	·50	·50
Correction for inequality of arms	·09	·04
Volume between the cylinders is	4412·87	4411·29

The mean of these is

$$4412{\cdot}08 \text{ c.c.}$$

which we take as the value of the volume between the cylinders.

If d is the mean distance between the cylinders, l the length of the inner cylinder, a and b the radii of the outer and inner cylinders respectively

$$d = \frac{4412{\cdot}08}{\pi\,(a+b)\,l};$$

so that

$$d = {\cdot}94128 \text{ cm.}$$

$$\frac{a}{b} = 1 + \frac{a-b}{b} = 1{\cdot}0800262;$$

$$2 \log \frac{a}{b} = {\cdot}15397063.$$

The thicknesses of the pieces of ebonite between the guard rings and the cylinder at the top and bottom were respectively ·2934 and ·288. Correcting for the air space the effective length of the cylinder is

$$60{\cdot}9784 + {\cdot}2907 = 61{\cdot}2691 \text{ cm.}$$

Hence the electrostatic measure of the capacity

$$= \frac{61{\cdot}2691}{{\cdot}15397063};$$

$$= \mathbf{397{\cdot}927} \text{ cm.}$$

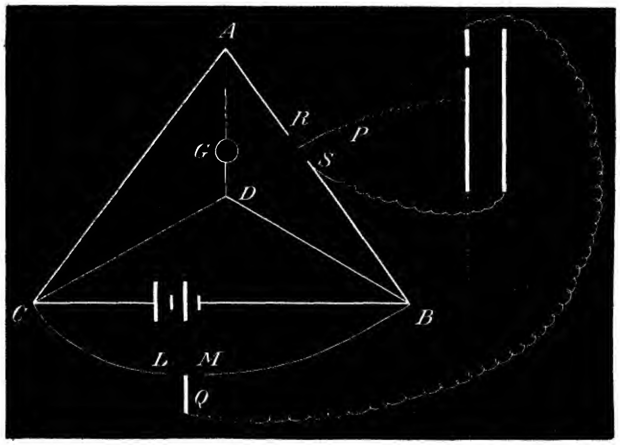
No correction is required for want of coincidence between the axes of the cylinders, for if c be the distance between the axes the correction is proportional to $c^2/(a^2 - b^2)$. In our experiments c was less than ·01, so that the correction only amounts to one part in more than 30,000.

The difference of potential between the guard ring and the cylinder depends upon the speed of the commutator, so that the correction on this account is made on the electromagnetic measure of the capacity.

The Electromagnetic Measure of the Capacity.

The arrangement employed in this measurement is represented in fig. 3.

Fig. 3.



ABCD is a Wheatstone's bridge with the galvanometer at G, and the battery between B and C. The arm AB is broken at R and S, which are two poles of a commutator, which alternately come into contact with a spring P, connected with the middle part of the inner cylinder of the condenser. The outer cylinder is connected to S. The points C and B are connected respectively with L and M, the two poles of a commutator, which alternately come into contact with a spring Q, attached to the guard ring of the condenser. The system is arranged so that when the commutators are working the order of events is as follows :—

I.	P on S.	Condenser discharged.
	Q on M.	Guard ring discharged.
II.	P on R.	Condenser begins to charge.
	Q on M.	
III.	P on R.	Condenser completely charged to potential (A)--(B).
	Q on L.	Guard ring charged to potential (C)--(B).
IV.	P on S.	Condenser begins discharging.
	Q on L.	
V.	P on S.	Condenser discharged.
	Q on M.	Guard ring discharged.

Thus, when the commutators are working, there will, owing to the flow of electricity to the condenser, be a succession of momentary currents through the galvanometer. The resistances are so adjusted that the effect of these momentary currents on the galvanometer just balances the effect due to the steady current, and there is no deflection of the galvanometer.

To investigate the relation between the resistances when this is the case, let us suppose that when the guard ring and condenser are charging

$$\dot{x} = \text{current through BC.}$$

$$\dot{y} = \text{current through AR.}$$

$$\dot{z} = \text{current through AD.}$$

$$\dot{w} = \text{currrent through CL.}$$

Thus, if a, b, α, β, γ are the resistances in the arms BC, AC, AD, BD, CD respectively, L the coefficient of self induction of the galvanometer, and E the electromotive force of the battery, we have

$$\mathrm{L}\ddot{z} + (b + \gamma + \alpha)\,\dot{z} + (b + \gamma)\,\dot{y} + \gamma\dot{w} - \gamma\dot{x} = 0 \quad . \quad . \quad . \quad . \quad (1)$$

$$(a + \gamma + \beta)\,\dot{x} - (\gamma + \beta)\,\dot{y} - \gamma\dot{z} - (\gamma + \beta)\,\dot{w} - \mathrm{E} = 0 \quad . \quad . \quad . \quad . \quad (2)$$

Now it is evident that the currents are expressed by equations of the following kind

$$\dot{x} = \dot{x}_1 + \dot{x}_2,$$
$$\dot{z} = \dot{z}_1 + \dot{z}_2,$$

where $\dot{x}_1$ and $\dot{z}_1$ express the steady currents when no electricity is flowing into the condenser, and $\dot{x}_2$, $\dot{z}_2$ are of the form $\mathrm{A}\epsilon^{-\lambda t}$, $\mathrm{B}\epsilon^{-\lambda t}$, and express the variable parts of the currents due to the charging of the condenser, $\dot{y}$ and $\dot{w}$ will be of the form $\mathrm{C}\epsilon^{-\lambda t}$, $\mathrm{D}\epsilon^{-\lambda t}$; t in all these equations is the time which has elapsed since the condenser commenced to charge.

Equations (1) and (2) will thus contain constant terms, and terms multiplied by $\epsilon^{-\lambda t}$, the latter must separately vanish, hence we have

$$\mathrm{L}\ddot{z}_2 + (b + \gamma + \alpha)\,\dot{z}_2 + (b + \gamma)\,\dot{y} + \gamma\dot{w} - \gamma\dot{x}_2 = 0 \,. \quad . \quad . \quad . \quad . \quad (3)$$

$$(a + \gamma + \beta)\,\dot{x}_2 - (\gamma + \beta)\,\dot{y} - \gamma\dot{z}_2 - (\gamma + \beta)\,\dot{w} = 0 \,. \quad . \quad . \quad . \quad . \quad (4)$$

Let Z, X be the quantities of electricity which have passed through the galvanometer and battery respectively, in consequence of the charging of the condenser, and

Y and W the charges in the condenser and guard ring. Then integrating equations (3) and (4), over a time extending from just before the condenser began to charge until it is fully charged, remembering that at each of these times $\dot{z}_2 = 0$, we get

$$(b + \gamma + \alpha) \mathrm{Z} + (b + \gamma) \mathrm{Y} + \gamma \mathrm{W} - \gamma \mathrm{X} = 0$$

$$(a + \gamma + \beta) \mathrm{X} - (\gamma + \beta) \mathrm{Y} - \gamma \mathrm{Z} - (\gamma + \beta) \mathrm{W} = 0,$$

hence eliminating X

$$\mathrm{Z}\left(b + \gamma + \alpha - \frac{\gamma^2}{a + \gamma + \beta}\right) + \mathrm{Y}\left(b + \gamma - \frac{\gamma(\gamma + \beta)}{a + \gamma + \beta}\right) + \mathrm{W}\gamma \frac{a}{a + \gamma + \beta} = 0.$$

In our experiments, the battery resistance is very small, being less than 1 ohm, while β is 500,000 ohms, b 200,000 ohms, and γ 3000 ohms, thus the third term is less than $\frac{1}{5,000,000}$th part of the second, and may be neglected, and we get, neglecting the battery resistance

$$\mathrm{Z} = - \frac{b}{b + \gamma + \alpha - \dfrac{\gamma^2}{\gamma + \beta}} \mathrm{Y}.$$

If $\{A\}$ $\{B\}$ $\{D\}$ denote the potentials of A, B, D when the condenser is fully charged, C the capacity of the condenser, then

$$\mathrm{Y} = \mathrm{C}[\{\mathrm{A}\} - \{\mathrm{B}\}].$$

But

$$\frac{\{\mathrm{A}\} - \{\mathrm{B}\}}{\alpha + \beta \dfrac{(b + \alpha + \gamma)}{\gamma}} = \frac{\{\mathrm{A}\} - \{\mathrm{D}\}}{\alpha}.$$

The right-hand side of this equation is $\dot{z}_1$, the steady current through the galvanometer, so that

$$\mathrm{Y} = - \mathrm{C}\dot{z}_1 \left(\alpha + \beta \frac{(b + \alpha + \gamma)}{\gamma}\right). \quad \text{(5)}$$

$$\mathrm{Z} = - \dot{z} b \mathrm{C} \frac{\left\{\alpha + \beta \dfrac{(b + \alpha + \gamma)}{\gamma}\right\}}{b + \gamma + \alpha - \dfrac{\gamma^2}{\gamma + \beta}}. \quad \text{(6)}$$

If the condenser is charged n times per second, the quantity of electricity which passes in consequence through the galvanometer per second is $n\mathrm{Z}$. If the galvanometer

needle remains undeflected, the quantity of electricity which passes through the galvanometer in unit time must be zero. But this quantity is $n\mathrm{Z} + \dot{z}_1$, so that

$$n\mathrm{Z} + \dot{z}_1 = 0.$$

Substituting this relation in equation (6), we get

$$\mathrm{C} = \frac{1}{n}\frac{\gamma}{b\beta}\frac{\left\{1 - \dfrac{\gamma^2}{(\gamma + \beta)(b + \gamma + \alpha)}\right\}}{1 + \dfrac{\gamma\alpha}{(b + \alpha + \gamma)\beta}} . \qquad (7)$$

From this equation, if we know the resistances and the speed, we can calculate the capacity.

In order to apply the correction for the difference of potential between the guard ring and the inner cylinder, we require $\{A\} - \{C\}/\{A\} - \{B\}$, now

$$\frac{\{A\} - \{C\}}{\{A\} - \{B\}} = \frac{b}{\alpha} \cdot \frac{\{A\} - \{D\}}{\{A\} - \{B\}}$$

$$= \frac{b}{\alpha + \beta\dfrac{(b + \alpha + \gamma)}{\gamma}} .$$

Where n was equal to 64 the approximate values of the resistances were

$$\begin{aligned} b &= 200{,}000 \text{ ohms.} \\ \alpha &= 20{,}000 \quad ,, \\ \beta &= 500{,}000 \quad ,, \\ \gamma &= 3{,}000 \quad ,, \end{aligned}$$

substituting the values

$$\frac{\{A\} - \{C\}}{\{A\} - \{B\}} = \tfrac{1}{183}.$$

We shall now go on to discuss the details of the method whose theory we have just given.

The Commutator.

The general view of this is shown in figs. (4), (5), (6). The framework is strongly made of cast iron, and somewhat resembles the headstock of a lathe, it is provided with two hardened steel centres, capable of adjustment, between which runs the axle of the commutator. This was made of tool steel but not hardened. The centres were very good, and the apparatus ran with very little friction and wear.

We will now describe the different pieces fixed to the axle beginning from the left of fig. 4.

Fig. 4.

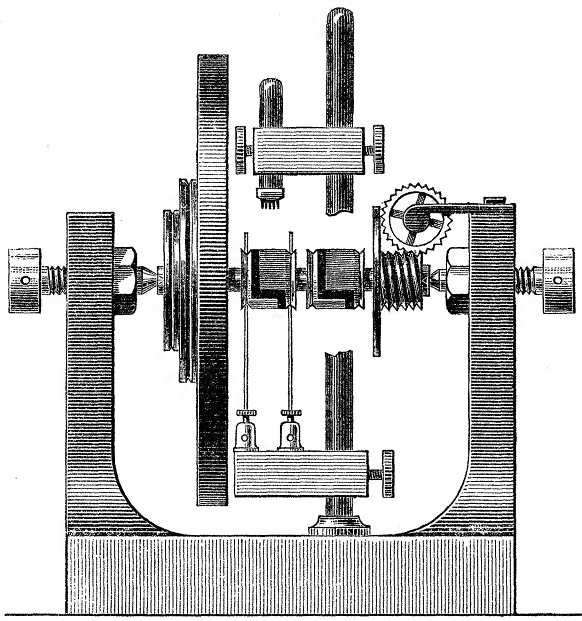
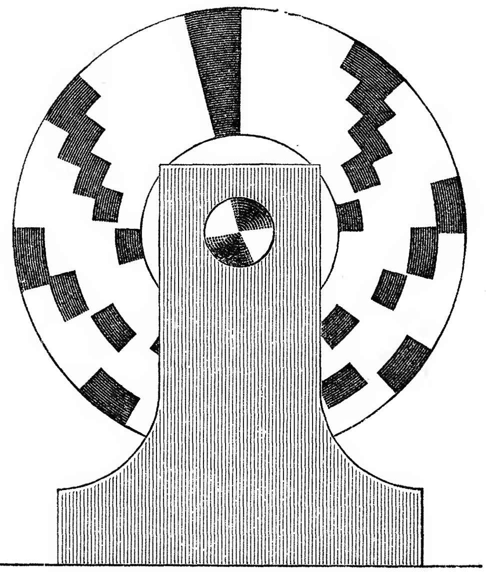


Fig 5.



0 5 10 15 20

SCALE OF CENTIMETRES.

First there is a wooden disc with two grooves for the driving string, next comes the wooden disc on which the stroboscopic pattern is painted. The end view of this is shown in fig. 5.

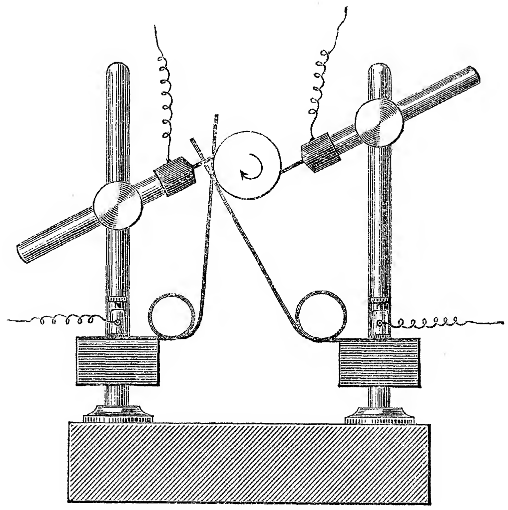
Beyond this are the two commutators, which are exactly alike, each is made of two portions of brass tube fixed to an ebonite bush. Two grooves are turned in the continuous portion of the tube, in which rest two wires by means of which electrical connection is made with the parts of the commutator. Although the slits in the commutator were not more than about 2 mm. wide yet no trouble at all was experienced through want of insulation. The commutator could be easily cleaned by scraping the ebonite between the parts of the commutator with a sharp tool. There was also very good insulation between the commutator and the axle. The insulation was tested several times by means of the gold leaf electroscope, and was found practically perfect.

Following the two commutators is an ebonite disc to prevent oil from the centres getting to the commutators.

The last thing on the axle is an endless screw in which gears a worm-wheel of 30 teeth. This wheel is furnished with a pin which makes contact with a spring once in every revolution of the wheel, *i.e.*, every 30 revolutions of the commutator. The spring is insulated from the framework, and the contact of the pin with it completes the circuit required for exciting one of the electromagnets of the recording apparatus. The spring is not shown in the figure.

The supports for the contact wires of the commutator and for the brushes were two pillars screwing into the base of the framework. One of these is shown in the figure, it is shown broken, and the two parts separated in order to show both parts of the commutator. Fig. 6 shows the arrangement as it would look in an end view if the

Fig. 6.



stroboscopic disc were absent. The wires for making contact with the commutator are clamped under the two binding screws and are made with a curl Q so as to be very elastic and to ensure contact with the commutator. Before the curl was put into the wires it was found that the jarring of the apparatus gradually caused the wires (though made of hard drawn brass) to relax their pressure on the commutator and to make uncertain contacts. With the curled wires no trouble was found in this respect, and the pressure could be made much less, saving both friction and wear. The binding screws clamping the wires are fixed to a piece of ebonite which can slide on the pillar and be fixed in any position by a screw. The pillar also carries a support for the brush. The brush itself is made of very fine hard drawn brass wire, soldered into a brass piece of a suitable shape. The way in which the brush acted depended a good deal upon the regularity and straightness of its wires. We found it best not to have a thick bunch of wire, but a thin layer only a few wires thick. The brass piece into which the brush wires were soldered, fits on the end of an ebonite rod passing through the brush holder and capable of being fixed in its proper position by means of a screw.

The whole arrangement was clamped down to a thick iron slab, resting on a strong table. In this way the vibrations, which would otherwise have been set up, were avoided.

The tuning fork, by means of which the speed was observed and regulated, was placed on a separate table. At first the two were on the same table, but it was found that at the speed at which the commutator made one revolution for each

complete vibration of the fork, the vibrations set up by the fork were sufficient to make the contacts uncertain. As finally fixed, the apparatus worked extremely satisfactorily, and would run for several hours without either the brushes or the springs requiring any adjustment.

The commutator was driven from a water motor which was supplied from a tank at the top of the laboratory to secure a constant head of water. It was driven by a band of fine fishing line joined with a "long splice"; any rougher method of joining produced a joint, the effect of whose passage over the pulley of the commutator was plainly seen by the observer at the galvanometer. A second band went from a small pulley on the motor to a pulley fixed within easy reach of the observer stationed at the tuning fork. The regulation of the speed was done by letting the auxiliary band run through the fingers, and slightly pressing it. This was found to be a much better plan than regulating by the band driving the commutator. But, in spite of this, the speed of the commutator as judged by the steadiness of the pattern seen through the slits of the tuning fork was subject to incessant small agitations, and it required considerable vigilance on the part of the observer at the fork to keep the pattern quite at rest. A heavier disc on the commutator would no doubt have made this easier.

The supply of water was so adjusted that it was able to drive the commutator slightly faster then the speed required. The necessary fine adjustment was made by slightly pressing the regulating band.

Determination of the Speed of the Commutator.

To ascertain the speed at which the commutator was being driven its stroboscopic disc was observed through a pair of narrow slits fastened to the prongs of an electrically maintained fork. This fork made approximately 64 complete vibrations per second. The disc was provided with circles containing 4, 5, 6, 7, 8 spots at equal intervals, so that when a distinct pattern was observed through the slits on the fork the commutator made one of the following numbers of revolutions per second:—

16·0, 18·3, 21·35, 25·6, 32·0, 36·6 42·7, 48·0,
51·2, 54·9, 64, 73·2, 76·9, 80·1, 85·4.

These numbers are respectively $\frac{1}{4}$, $\frac{2}{7}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{4}{7}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{6}{7}$, 1, $\frac{8}{7}$, $\frac{6}{5}$, $\frac{5}{4}$, and $\frac{4}{3}$ of 64.

Any one of these speeds could be obtained by simply regulating the supply of water to the motor. Higher speeds could, of course, have been observed, but the motor would not drive the commutator much faster than 80 revolutions per second.

Experiments at most of these speeds will be found below.

It will be observed that this method gives a great choice of speeds, all of which can be determined with the same accuracy, and whose relation one to another is known with absolute accuracy.

The standard to which the speed was referred during the experiments was the

standard fork used by Lord RAYLEIGH. This makes about 128 complete vibrations per second. The electrically driven fork maintained another fork whose natural period is about half its own. This gave beats with the standard, and by counting the beats the speed of the fork through which the stroboscopic disc was observed could be determined in terms of that of the standard fork.

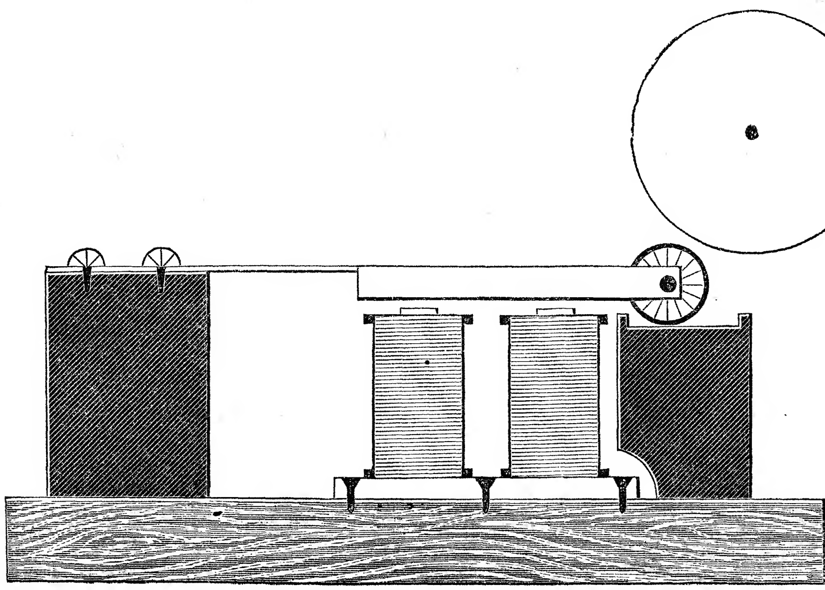
Determination of the Speed of the Standard Fork.

It was considered advisable to make a new determination of the standard fork. Lord RAYLEIGH ('Phil. Trans.,' 1883, p. 320) found that its speed at $t°$ C was

$$128{\cdot}140\ \{1 - (t - 16) \times {\cdot}00011\}.$$

We have taken Lord RAYLEIGH's value of the temperature coefficient.

We determined the speed of the standard in the following way. At one end of the commutator an endless screw was fixed with a cog-wheel geared into it. The cog-wheel makes one revolution for 30 made by the commutator. A pin fixed to the wheel touches a spring once in every revolution, and completes an electric circuit



which causes a mark to be made on the tape of the recording apparatus. The laboratory clock is also arranged so as to complete a circuit once every second. A paper tape is pulled along at an approximately uniform rate between guides on a block of wood. Two electro-magnets are fixed at right angles to the guides. One of these is excited when contact is made by the cog-wheel of the commutator, the other when contact is made by the clock. The armatures are kept away from their magnets by springs fixed to one end.

The other end of the armature carries a fork in which a small disc can rotate. When the magnet is not excited the disc presses against a roller covered with printer's ink. When the magnet is excited the disc presses on the paper tape as it passes underneath it. The disc is kept inked by revolving the inking roller.

The general disposition of the apparatus will be seen from the figure.

As the tape is drawn along we get two series of marks, one due to the commutator, the other to the clock. By comparing the position of the marks made by the commutator with those made by the clock we get a very accurate method of timing the commutator. We can find the time at which say 15 commutator contacts are made, and then, after the lapse of 5 or 10 minutes, find the time at which another 15 commutator contacts are made. Subtracting the one set of times from the others we get 15 intervals which should, of course, be equal. Taking the average we get a very accurate value for the time occupied by (say) 600 revolutions of the cog wheel.

The paper tape was at first drawn along by a Morse receiver, but as this did not act very uniformly it was replaced by a small winding arrangement driven by the same motor as the commutator. This ran perfectly uniformly.

The method of experimenting was as follows: when the observing forks had been running for a few minutes, so that everything had got steady, the beats between the standard fork and the auxiliary fork were counted. The forks were so arranged that 20 beats occurred in about 65 seconds. The motor was then started and regulated so that the commutator could be kept at half the speed of the fork, *i.e.*, about 32 revolutions per second.

The recording apparatus was then started and allowed to run for 5 or 10 minutes, the commutator being kept at a constant speed by observing the stroboscopic pattern through the fork. The apparatus was then stopped and the beats again counted.

Our reason for adopting this method was that we had originally intended to measure the speed of the commutator while we were making the electrical observations. This was, however, found to be too laborious, and had to be given up. The Laboratory clock was compared with the clock belonging to the Cambridge Philosophical Society, which is regularly rated from the Observatory.

The last three observations, when the apparatus was working very satisfactorily give for the rate of the standard fork at 16° C.

December 19 (tape running for 6 minutes) . . .	128·1081
February 14 (,, ,, 10 ,,) . . .	128·0909
,, 15 ,, ,, ,, ,, . . .	128·1146
Mean	128·1045

Thus according to our observations the fork is slightly slower than when used by Lord RAYLEIGH. This is what might be expected from the secular softening of the steel.

The Galvanometer.

This was one made in the Laboratory. It has two coils, each with about 16,000 turns of wire. The resistance of the coils when in series is 17,380 legal ohms. Great care was taken with the insulation, which, when tested by means of a gold leaf electroscope, was found to be practically perfect.

The Battery.

We used 36 very small storage cells, two sets of 18 being placed in parallel. The battery had thus an electromotive force of about 36 volts. The small size of the battery enabled us to insulate it with ease. The insulation was tested by a gold leaf electroscope, but no leak could be detected.

The Resistances.

The resistances used were contained in three boxes.

I. A Wheatstone's bridge box, No. 1256, ELLIOTT. Legal ohms, with coils ranging from 1 to 5000 ohms, and proportional arms, each containing 10, 100, 1000 ohms.

II. A box by ELLIOTT, containing 4 coils, 10,000, 20,000, 30,000, 40,000 B.A. units.

III. A box by MUIRHEAD, containing originally 10 coils, each 100,000 B.A. units.

The box I. was provided with an aperture for a thermometer. The other boxes had no such provision, but as they were always kept permanently on the same table as box I. we may hope that their temperature did not differ much from that of I.

The resistances of the coils in II. and III. were always ascertained by means of I. It was, therefore, necessary to obtain some definite knowledge of I. with reference to some coils whose values are accurately known.

The standard coils used were

No. 141, C.L.C. No. 102	. . .	10·00103 legal ohms at	16°·7 C.
No. 143, C.L.C. No. 104	. . .	99·9977 ,, ,,	16°·05 C.
No. 145, C.L.C. No. 106	. . .	1000·306 ,, ,,	17°·4 C.

The Legal ohm being assumed to be 1/·9889 B.A. unit.

See 'British Association Report,' 1885.

Mr. GLAZEBROOK informs us that the temperature coefficient of these coils may be taken as ·0003 per 1° C. We shall take the same temperature coefficient for the coils in the boxes.

Calling the coils in the "proportional arm" which joins the rest of the box 10_a, 100_a, 1000_a, those in the other arm 10_b, 100_b, 1000_b we found

$$\frac{1000_b}{1000_a} = 1 - \frac{\cdot 3}{10000}$$

$$\frac{1000_b}{100_a} = 10\left\{1 - \frac{1\cdot 8}{10000}\right\}$$

$$\frac{1000_b}{10_a} = 100\left\{1 + \frac{\cdot 76}{10000}\right\}.$$

Correcting for the inequalities of 1000_a and 1000_b we found by comparison with the Standard coils the following values at 16° C., for the various coils in box I.

	Legal ohms.
100_α	100·144
100_β	100·144
100	100·113.

The last resistance was made up from all the coils from 1 to 50,

	Legal ohms.
1000_α	1001·06
1000_β	1001·01
1000	1001·06.

The last was made up of all the coils from 1 to 500,

	Legal ohms.
2000	2002·22
5000	5004·88.

We see from this that the coils are very consistent, and on the average greater than legal ohms, in the ratio of 1·0011 to 1.

The box II. was measured as a whole, using the "proportional" coils 1000_b and 10_a. The apparent value was 98765. Hence the true value is

$$98765 \times 1\cdot 0011\left(1 + \frac{\cdot 76}{10000}\right) = 98870 \text{ legal ohms at } 16^\circ \text{ C.}$$

The coils of box III. were tested in the same way and were found to be

	Resistance as measured by box I.	True resistance in legal ohms.
1.	98723	98878
2.	98617	98732
3.	98690	98805
4.	98727	98842
5.	98717	98832
6.	98768	98883

The box III. had one of its coils wrong when we began to use it, and during part of our work when we were attempting to use much greater battery power than we finally adopted, three more of the coils gave way. For this reason we could only use six out of the ten coils. These six coils, as well as box II., kept their resistances very constant during the whole of the investigation. They were tested several times during a whole year, and no perceptible change was detected.

The resistance boxes were placed on blocks of paraffin, and the insulation tested by a gold leaf electroscope.

Method of making the Observations.

The battery, resistance boxes, galvanometer, and condenser were connected as in the diagram (3), the commutator being placed so near to the condenser that a very short wire sufficed to make the connection. The insulation of the whole system when connected up was tested from time to time by a gold leaf electroscope.

The electrically driven fork was set going and the beats of the auxiliary fork with the standard fork observed. The water supply was then adjusted so that the motor drove the commutator at the required speed. One observer (G. F. C. S.) observed the stroboscopic disc of the commutator through the slits of the fork, and kept the speed steady by means of the controlling arrangement already described. The other observer (J. J. T.) observed the galvanometer. When the speed of the commutator had got steady, resistances were taken out of the Wheatstone's bridge box in the arm CD until no deflection was produced when the galvanometer circuit was broken. The sensitiveness of the galvanometer was such that the effect produced by altering the resistance in CD by 2 ohms in 3000 could be detected, and the speed of the commutator was kept so steady that the light reflected from the galvanometer mirror did not move over more than half a division; the deflection produced by the condenser when not balanced was more than 500 scale divisions.

The battery was then reversed and the operation repeated. The wire connecting the condenser to the commutator was then detached from the condenser, and the same operation repeated. In this way the capacity of the wire was determined. The temperatures of the fork and resistances were then read, and the beats of the auxiliary fork with the standard fork again determined.

The results of these observations are exhibited in the annexed Table.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Date.	Speed.	Beats per minute.	Temperature of fork.	b.	β.	Value of γ for condenser + wire.	Value of γ for wire.	Value of γ for condenser.	Reduced to 64.	Temperature of resistances.	Correction to 16° for resistances.	Correction to 21 beats per minute.	Correction to 16° for fork.	Correction to $n\mathrm{C} = \frac{\gamma}{b\beta}$.	Correction to legal ohms.	Correction for difference of potential between g. r. and condenser.	Corrected value of γ.	Electromagnetic measure of capacity, $\times 10^{-21}$.
Dec. 17	64	21·4	17·5	197753	494089	2797·5	39·5	2758	2758	16·5	− ·41	+ ·14	+ ·45	− 1·37	+ 3·04	+ 1·72	2761·6	443·448
,,	32	21·4	17·5	,,	,,	1399·5	20	1379·5	2759	16·5	− ·41	+ ·14	+ ·45	− ·68	+ 3·04	+ ·86	2762·4	443·576
,,	48	21·5	17·5	,,	,,	2099	30	2069	2758·7	16·5	− ·41	+ ·18	+ ·45	− 1·07	+ 3·04	+ 1·29	2762·1	443·528
,,	80	21·5	17·5	,,	,,	3498	50·25	3447·75	2758·2	16·5	− ·41	+ ·18	+ ·45	− 1·80	+ 3·04	+ 2·15	2761·8	443·480
,,	64	21·8	18	,,	,,	2798	40·5	2757·5	2757·5	17·2	− 1	+ ·29	+ ·60	− 1·37	+ 3·04	+ 1·72	2760·8	443·319
,,	55	21·8	18	,,	,,	2399	34·5	2364·5	2758·6	17·2	− 1	+ ·29	+ ·60	− 1·15	+ 3·04	+ 1·49	2761·9	443·496
,,	42	21·8	18	,,	,,	1866	27	1839	2758·5	17·2	− 1	+ ·29	+ ·60	− ·91	+ 3·04	+ 1·14	2761·6	443·448
Dec. 18	64	21·7	17·5	296585	395257	3355	49	3306	3306	17·2	− 1·2	+ ·29	+ ·54	− 1·66	+ 3·64	+ 3·20	3310·8	443·112
,,	48	21·7	17·5	,,	,,	2517·5	37·5	2480	3306·7	17·2	− 1·2	+ ·29	+ ·54	− 1·32	+ 3·64	+ 2·40	3311·1	443·153
,,	32	21·7	17·5	,,	,,	1679·5	25	1654·5	3309	17·2	− 1·2	+ ·29	+ ·54	− ·84	+ 3·64	+ 1·60	3313·0	443·407
,,	80	21·7	17·5	,,	,,	4193	60·5	4132·5	3306	17·4	− 1·4	+ ·29	+ ·54	− 2·38	+ 3·64	+ 4·00	3310·2	443·032
,,	64	21·8	18·2	,,	,,	3359·5	49	3310·5	3310·5	17·2	− 1·2	+ ·35	+ ·79	− 1·66	+ 3·64	+ 3·20	3315·6	443·754
,,	55	21·8	18·2	,,	,,	2881·5	43	2838·5	3311·6	17·5	− 1·5	+ ·35	+ ·79	− 1·54	+ 3·64	+ 2·74	3315·7	443·768
,,	48	21·8	18·2	,,	,,	2520	38	2482	3309·3	18	− 2	+ ·35	+ ·79	− 1·32	+ 3·64	+ 2·40	3313·2	443·434
,,	32	21·6	18·2	,,	,,	1680·5	25	1655·5	3311	18	− 2	+ ·26	+ ·79	− ·84	+ 3·64	+ 1·60	3314·4	443·594
,,	16	21·6	18·2	,,	,,	841	13	828	3312	18	− 2	+ ·26	+ ·79	− ·41	+ 3·64	+ ·80	3315·0	443·675
,,	80	21·6	18·2	,,	,,	4196	61·5	4134·5	3307·6	18·1	− 2·1	+ ·26	+ ·79	− 2·38	+ 3·64	+ 4·00	3311·8	443·246
Dec. 20	64	21·6	16·6	197748	494094	2799	41·5	2757·5	2757·5	16·2	− ·16	+ ·22	+ ·18	− 1·37	+ 3·04	+ 1·72	2761·2	443·390
,,	32	21·8	16·8	,,	,,	1399·5	21	1378·5	2757	16·4	− ·33	+ ·29	+ ·24	− ·68	+ 3·04	+ ·86	2760·4	443·262
,,	48	21·8	16·8	,,	,,	2101	30·75	2070·25	2760·3	16·4	− ·33	+ ·29	+ ·24	− 1·07	+ 3·04	+ 1·29	2763·7	443·792
,,	80	21·6	16·6	,,	,,	3500	51·5	3448·5	2758·8	16·4	− ·33	+ ·22	+ ·18	− 1·80	+ 3·04	+ 2·15	2762·2	443·551
,,	55	21·6	16·6	,,	,,	2401·5	35·25	2366·25	2760·6	16·4	− ·33	+ ·22	+ ·18	− 1·15	+ 3·04	+ 1·49	2764·0	443·840
,,	64	21·6	16·6	,,	,,	2800·5	41·3	2759·2	2759·2	16·7	− ·57	+ ·22	+ ·18	− 1·37	+ 3·04	+ 1·72	2762·4	443·583

Explanation of Table of Results and Methods of Reduction.

Column 2 gives the approximate speed of the commutator.

Column 3 gives the beats per minute between the standard fork and the auxiliary fork driven by the electrically maintained fork. The auxiliary fork vibrated twice as fast as the driving fork, and was slightly slower than the standard.

Column 4. The temperature of the fork given by a thermometer hung a short distance from the fork.

Columns 5 and 6. The resistances of the arms AC and BD in legal ohms at 16° C.

Column 7 gives the value of the resistance in CD required for the balance when both the condenser and the connecting wire were joined to the key of the commutator. We shall denote this by γ_1. Each number is the mean of two observations made with the current from the battery flowing first in one direction and then in the opposite. The difference between the two readings very seldom amounted to more than 1 ohm.

Column 8 gives the value of the resistance in CD required for the balance when the wire alone is joined to the key of the commutator. We shall denote this by γ_2. Each is the mean of two readings corresponding to the two directions of the battery current.

Column 9 gives the difference between the last two columns. We must remark that, since the formula $n\mathrm{C} = \gamma/\beta b$ is not sufficiently accurate for our purpose, that this difference does not strictly represent the value of γ, which would be required to balance the condenser alone. With the resistances employed we may write the formula (7)

$$n\mathrm{C} = \frac{\gamma}{\beta b}\left\{1 - \frac{\gamma(\gamma + \alpha)}{(\alpha + b)\beta}\right\},$$

so that if, as in our case, we keep β and b constant, $n\mathrm{C}$ is not quite proportional to γ. Strictly, we should find the capacity of the combination of the condenser and wire, and then subtract the capacity of the wire. What we have done is to calculate as if $\gamma_1 - \gamma_2$ represented the capacity of the condenser. The difference amounts to writing in the small term $\gamma(\gamma + \alpha)/(\gamma + b)\beta$, $\gamma_1 - \gamma_2$ instead of γ_1, since the correcting term inside the bracket, in the case of the wire, is too small to be appreciable. Since, however, this term is already very small, and does not affect the result by much more than one part in 2000, and the change we have made only alters its value by 1 per cent. at most, it is evident that it will produce no appreciable effect on the value of the capacity.

Column 10 is headed "(reduced to 64)." Since the value of the capacity is given by

$$\mathrm{C} = \frac{\gamma}{n\beta b}\left\{1 - \frac{\gamma(\gamma + \alpha)}{(\alpha + b)\beta}\right\},$$

or with sufficient accuracy for our purpose by

$$C = \frac{\gamma}{nb\beta}\left\{1 - \frac{\gamma\alpha}{b\beta}\right\};$$

and if γ_a, γ_b are the values of γ corresponding to the speeds n_a, n_b, then

$$\frac{\gamma_a}{n_a}\left\{1 - \frac{\gamma_a\alpha}{b\beta}\right\} = \frac{\gamma_b}{n_b}\left\{1 - \frac{\gamma_b\alpha}{b\beta}\right\},$$

or

$$\gamma_a\left(1 - \frac{\gamma_a\alpha}{b\beta}\right) = \frac{n_a}{n_b}\gamma_b\left\{1 - \frac{\alpha}{b\beta}\gamma_b\right\}.$$

Hence we see that we may take as the corresponding uncorrected value of γ_a the value $\gamma_b n_a/n_b$, if we apply to this the correction $1 - \frac{\alpha}{b\beta}\gamma_b$ instead of the correction $\left(1 - \frac{\alpha}{b\beta}\gamma_a\right)$, this is what we have done in compiling this table, and this column contains the values of $\gamma_b n_a/n_b$.

Column 11 gives the temperature of the box from which the resistance was taken. The temperature of the other two boxes are assumed to be the same. Since they were always on the same table as the γ box, and the temperature of this box varied very little, this assumption will not lead to any serious error.

Column 12. "Correction to 16° for resistances" from the formula

$$nC = \frac{\gamma}{b\beta}$$

we see that if we take the temperature coefficients to be the same for the three resistances and equal to ·0003, then we may throw all the effect of the variation on γ if for each degree above 16 we *subtract* from γ ·0003 γ. In the first and third sets this amounts to ·82 ohm for each degree, in the second to 1 ohm per degree.

Column 13. "Correction to 21 beats per minute." The auxiliary fork always made rather more than 21 beats per minute with the standard. When making 21 beats the speed of the auxiliary fork is 127·7545 complete vibrations per second, and 1 additional beat per minute indicates a diminution in the speed in the ratio $1 - \frac{1}{128 \times 60}$ to 1. Since we wish to treat the observations as if the speed were constant we must for each additional beat increase γ in the ratio $1 + \frac{1}{128 \times 60}$ to 1, *i.e.*, in the first and third set observations we must add ·36 ohm to γ, and in the second set ·43 ohm.

Column 14. "Correction to 16° for fork." The fork has a temperature coefficient of − ·00011, so that if we regard the speed as constant we must increase γ in the proportion 1·00011 to 1 for each degree the temperature of the fork exceeds 16° C. In the first and third sets this amounts to ·30, and in the second set to ·36 ohm for each degree of excess of temperature above 16° C.

Column 15. "Correction to $nC = \gamma/b\beta$." We see from equation (7), page 606, that this is only an approximation to the correct value for nC, which is

$$nC = \frac{\gamma}{b\beta} \frac{\left\{1 - \frac{\gamma^2}{(\gamma + \beta)(b + \gamma + \alpha)}\right\}}{\left\{1 + \frac{\gamma\alpha}{(b + \alpha + \gamma)\beta}\right\}}$$

or with sufficient accuracy for our purpose

$$nC = \frac{\gamma}{b\beta}\left\{1 - \frac{\gamma(\gamma + \alpha)}{\beta(b + \alpha + \gamma)}\right\},$$

thus, if we wish to use the formula $nC = \gamma/b\beta$ we must diminish γ in the ratio $1 - \frac{\gamma(\gamma + \alpha)}{\beta(b + \alpha + \gamma)}$ to 1. The amount by which γ is to be diminished is given in column 15.

Column 16. "Correction to legal ohms." The values of b and β given in columns (5) and (6) are already expressed in terms of legal ohms, but the values of γ are expressed in terms of the resistances in the Wheatstone bridge box, which are greater than legal ohms in the proportion 1·0011 to 1. We have then to add 3·04 ohms to γ in the first and third sets of experiments, and 3·64 ohms in the second.

Column 17. "Correction for the difference of potential between the middle cylinder and the guard ring." By equation (6), page 9, if the difference of potential between the outer cylinder and the guard ring is less by δV than that between the outer cylinder and the inner cylinder, the capacity is greater than it would be if $\delta V = 0$ in the ratio

$$1 + \frac{\delta V}{V}\frac{h}{l}\left\{\frac{t}{c} - \frac{2}{\pi}\log_e\frac{4c}{h\epsilon}\right\} \text{ to } 1.$$

where V is the difference of potential between the cylinders, t the thickness of the guard ring, $2c$ the thickness of the pieces of ebonite, and h the distance between the cylinders.

Since $t = 1$, $h = 1$, $c = \cdot145$ and $\delta V/V = -b/\left\{\alpha + \beta\frac{(b + \alpha + \gamma)}{\gamma}\right\}$ the capacity is less than it would be if the guard ring and the cylinder were at the same potential in the ratio of

$$1 - \frac{7\cdot5}{61}\frac{b\gamma}{\alpha\gamma + \beta(b + \alpha + \gamma)} \text{ to } 1,$$

so that in order to get the corresponding value of γ when the guard ring and cylinder are at the same potential, we must add to γ

$$\frac{7\cdot5}{61}\frac{b\gamma^2}{\alpha\gamma + \beta(b + \alpha + \gamma)},$$

and it is this correction which is given in column 17.

Column 18 contains the values of γ which result when all these corrections are made.

Column 19 contains the values of the electromagnetic measure of the capacity calculated from the formula

$$C = \frac{\gamma}{n\beta b}.$$

Since the auxiliary fork makes 21 beats per minute with the standard fork, which makes 128·1045 vibrations per second, and since the observing fork makes half the number of vibrations of the auxiliary fork

$$n = 63{\cdot}8773.$$

Assuming that the B.A. unit $= {\cdot}9867 \times 10^9$ in absolute measure which corresponds to the legal ohm being $\cdot9978 \times 10^9$, we have

$$C = \frac{\gamma \times 10^{-9}}{b\beta \times 63{\cdot}8773 \times {\cdot}9977}$$

We have supposed that the resistance to the variable current which passes through the resistance coils during the charging of the condenser, is the same as the resistance to a steady current. This is justified by the following investigation. Though in our experiments the resistances were so large that the charging was not oscillatory, let us suppose that it is oscillatory and has the maximum frequency $1/\sqrt{\mathrm{LC}}$. Then if r' is the resistance to this oscillatory current, r the resistance to a steady current

$$r' = r\{1 + kn^2\},$$

where k is a small numerical coefficient and

$$n = \frac{\pi}{\sqrt{\mathrm{LC}}} \frac{a^2}{\sigma},$$

where a is the radius of the cross section of the wire, and σ its specific resistance. The coefficient of self induction of a galvanometer similar to the one we used was found some time ago to be 5×10^9, $\sigma = 2 \times 10^4$ and $a = {\cdot}023$ for 32 B.W.G.; substituting these numbers we find $n = 10^{-3}$ about, and

$$n^2 = 10^{-6},$$

so that the correction will only amount to one part in a million, and may be neglected.

The mean of the first set of observations			$= 443{\cdot}471 \times 10^{-21}$.
,,	second	,,	$= 443{\cdot}417 \times 10^{-21}$.
,,	third	,,	$= 443{\cdot}569 \times 10^{-21}$.
The mean of all the observations . .			$= 443{\cdot}486 \times 10^{-21}$.

The means of the observations at different speeds are given in the following table.

Speed.	Electromagnetic measure of capacity $\times 10^{-21}$.
80	443·327
64	443·434
55	443·701
48	443·478
42	443·448
32	443·460
16	443·675

The means for the different speeds thus agree very well together, the greatest difference from the mean being about one part in 2000. There does not seem any indication of an effect depending on the number of times the condenser is charged per second, such as was very marked in Professor ROWLAND's experiments ('Phil. Mag.,' vol. 28, 1889).

Since the electrostatic measure of the capacity is 397·927, and the electromagnetic measure $443{\cdot}454 \times 10^{-21}$

$$``v" = \sqrt{\left(\frac{397{\cdot}927}{443{\cdot}486 \times 10^{-21}}\right)}$$

$$= \mathbf{2{\cdot}9955 \times 10^{10}} \text{ cm. sec}^{-1}.$$

The value of the B.A. unit is taken to be $\cdot 9867 \times 10^{9}$ in absolute measure. This value of "v" agrees very nearly indeed with the value obtained by the most recent experiments for the velocity of light in air, these are

CORNU (1878)	$3{\cdot}003 \times 10^{10}$ cm. sec^{-1}.
MICHELSON (1879) . . .	$2{\cdot}9982 \times 10^{10}$.
MICHELSON (1882) . . .	$2{\cdot}9976 \times 10^{10}$.
NEWCOMB (1885) . . .	2·99615 ⎫
,, ,, . . .	2·99682 ⎬ $\times 10^{10}$.
,, ,, . . .	2·99766 ⎭

In conclusion we desire to express our thanks to Mr. R. S. COLE, of Emmanuel College, who has given us valuable assistance on several occasions.

The following table taken from Mr. E. B. ROSA's paper on the Determination of "v" ('Phil. Mag.,' vol. 28, 1889, p. 315), gives the results of previous determinations of "v":—

1856.	WEBER and KOHLRAUSCH	$3 \cdot 107 \times 10^{10}$.
1869.	W. THOMSON and KING	$2 \cdot 808 \times 10^{10}$.
1868.	MAXWELL	$2 \cdot 842 \times 10^{10}$.
1872.	M'KICHAN	$2 \cdot 896 \times 10^{10}$.
1879.	AYRTON and PERRY	$2 \cdot 960 \times 10^{10}$.
1880.	SHIDA	$2 \cdot 955 \times 10^{10}$.
1883.	J. J. THOMSON	$2 \cdot 963 \times 10^{10}$.
1884.	KLEMENČIČ	$3 \cdot 019 \times 10^{10}$.
1888.	HIMSTEDT	$3 \cdot 009 \times 10^{10}$.
1889.	W. THOMSON	$3 \cdot 004 \times 10^{10}$.
1889.	E. B. ROSA	$2 \cdot 9993 \times 10^{10}$.